

Statistical mechanics of the vacuum

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Abstract

The vacuum is full of virtual particles which exist for short moments of time. In this paper we construct a chaotic model of vacuum fluctuations associated with a fundamental entropic field that generates an arrow of time. The dynamics can be physically interpreted in terms of fluctuating virtual momenta. This model leads to a generalized statistical mechanics that distinguishes fundamental constants of nature.

1 Introduction

Statistical mechanics has been applied to a large variety of complex systems, with short-range and long-range interactions and many different underlying microscopic dynamics. Still what has never been done is to develop a kind of statistical mechanics formalism for the vacuum itself. Surely the vacuum, according to Heisenberg's uncertainty principle, contains a large number of virtual particles that exist for short moments of time. Hence there is a large number of fluctuating momentum and position variables, which a priori should allow for a statistical mechanics description, at least in a generalized sense.

In this paper we consider a simple model that could be regarded as a first step towards such a statistical mechanics description of spontaneous momentum fluctuations in the vacuum. On a microscopic level, our approach will lead to a so-called chaotic string dynamics [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13]. This is an additional dynamics to the standard model field equations—basically the dynamics of an entropy producing chaotic field. This field is not directly measurable but evolves within the bounds set by the uncertainty relation.

Our model of the vacuum is interesting for two reasons: Firstly, an arrow of time is naturally produced by this model, since the underlying chaotic dynamics corresponds to a coupled Bernoulli shift of information which distinguishes one direction of time. Secondly, certain observables associated with chaotic strings have been previously shown [1, 2] to distinguish standard model parameters as corresponding to states of minimum vacuum energy. In that sense the generalized statistical mechanics model considered has the scope to fix fundamental constants by first principles.

2 Fluctuations of momenta and positions

Let us construct a probabilistic model of vacuum fluctuations. Consider an arbitrary spatial direction in 3-dimensional empty space, described by a unit vector \vec{u} . We know that the vacuum is full of virtual particle-antiparticle pairs, for example e^+e^- pairs. Consider virtual momenta of such particles, which exist for short time intervals due to the uncertainty relation. From quantum mechanics (indeed, already from 1st quantization) we know that the phase space is effectively divided into cells of size of $O(\hbar)$. The uncertainty

principle implies

$$\Delta p \Delta x = O(\hbar). \quad (1)$$

Here Δp is some momentum uncertainty in \vec{u} direction, and Δx is a position uncertainty in \vec{u} direction. In the following, we will regard Δp and Δx as random variables. A priori we do not know anything about the dynamics of these random variables. But three basic facts are clear:

- (i) Δp and Δx are not independent. Rather, they are strongly correlated: A large Δp implies a small Δx , and vice versa.
- (ii) There are lots of virtual particles in the vacuum. For each of those particles, and for each of the directions of space, there is a momentum uncertainty Δp^i and a position uncertainty Δx^i , making up a phase space cell i .
- (iii) If the particle of a virtual particle-antiparticle pair has momentum Δp^i , then the corresponding antiparticle has momentum $-\Delta p^i$ in the rest frame.

We label the phase space cells in \vec{u} -direction by the discrete lattice coordinate i , taking values in \mathbf{Z} . With each phase space cell i we associate a rapidly fluctuating scalar field Φ_n^i . It represents the momentum uncertainty $\Delta p = p_n^i$ in \vec{u} direction in cell i at time $t = n\tau$ in units of some maximum momentum p_{max} :

$$p_n^i = p_{max} \Phi_n^i \quad \Phi_n^i \in [-1, 1] \quad (2)$$

τ is some appropriate time unit. We may call p_n^i the ‘spontaneous momentum’ associated with cell i at time n . The particle and antiparticle associated with cell i have momentum p_n^i and $-p_n^i$.

Let us introduce a position uncertainty variable x_n^i by writing

$$p_n^i x_n^i = \Gamma^{-1} \hbar \quad (3)$$

Here Γ is a constant of order 1. Indeed, eq. (3) states that the position uncertainty random variable x_n^i is essentially the same as the inverse momentum uncertainty random variable $1/p_n^i$, up to a constant times \hbar . By this we certainly realize property (i). The variable x_n^i is related to the rapidly fluctuating field Φ_n^i by

$$x_n^i = \frac{\hbar}{\Gamma p_{max} \Phi_n^i} \quad (4)$$

By definition, the sign of x_n^i is equal to the sign of the field Φ_n^i .

The phase space cells, labeled by the index i , can be represented as intervals of constant length \hbar/Γ . Each phase space cell is 2-dimensional (1 momentum, 1 position coordinate). Although the volume of the cells is constant, the shapes of the phase space cells fluctuate rapidly, since (in suitable units) their side lengths are given by Φ_n^i and $1/\Phi_n^i$. Indeed, p_n^i and x_n^i fluctuate both in time n and in the lattice direction i , and are uniquely determined by the random variable Φ_n^i .

It is clear that the Φ_n^i must have strong stochastic or chaotic properties in order to serve as a good model for vacuum fluctuations. On the other hand, since no physical measurements are able to determine the precise momentum and position within a phase space cell (due to the uncertainty relation), the dynamics of the Φ_n^i is a priori unknown. We cannot measure a concrete time sequence of vacuum fluctuations. Nevertheless, we are able to measure *expectations* of vacuum fluctuations.

3 Newton's law and self interaction

Let us now introduce a dynamics for the field Φ_n^i . It will ultimately lead to a coupled map lattice, the so-called chaotic string dynamics [1, 2], which is underlying our statistical mechanics of the vacuum at a microscopic level. It should be clear that the dynamics itself is not observable, due to the uncertainty relation, but that expectations with respect to the dynamics should be measurable in experiments. We attribute to each phase space cell i a self-interacting potential that generates the dynamics. For example, we may choose a ϕ^4 -theory, where the potential is of the form

$$V(\Phi) = \left(\frac{1}{2}\mu^2\Phi^2 + \frac{1}{4}\lambda\Phi^4 \right) mc^2 + C. \quad (5)$$

In our physical interpretation Φ is dimensionless, m is of the order of the mass of the virtual particles under consideration, μ^2 and λ are dimensionless parameters, and C is an additive constant. The 'force' in \vec{u} -direction due to this self-interacting potential is given by

$$F(\Phi) = -\frac{1}{c\tau} \frac{\partial}{\partial \Phi} V(\Phi) = (-\mu^2\Phi - \lambda\Phi^3) \frac{mc}{\tau} \quad (6)$$

(the factor $1/(c\tau)$ is needed for dimensional reasons, regarding t as physical time). We assume that the change of momentum is given by Newton's law

$$\frac{\partial}{\partial t}p = p_{max} \dot{\Phi} = F(\Phi). \quad (7)$$

Note that Newton's law is also valid in a relativistic setting, provided t denotes proper time.

The smallest time unit of our model of vacuum fluctuations is τ . This means that it does not make sense to consider Newton's law for infinitesimally small time differences Δt , since these would yield infinite energies ΔE , from $\Delta E \Delta t = O(\hbar)$. Thus we write eq. (7) in the finite-difference form

$$p_{max} \frac{\Phi_{n+1} - \Phi_n}{\tau} = (-\mu^2 \Phi_n - \lambda \Phi_n^3) \frac{mc}{\tau}. \quad (8)$$

It is remarkable that the unknown time lattice constant τ drops out. For arbitrary τ we get a dynamics that is given by the cubic mapping

$$\Phi_{n+1} = \left(1 - \frac{\mu}{\nu}\right) \Phi_n - \frac{\lambda}{\nu} \Phi_n^3, \quad (9)$$

where $\nu := \frac{p_{max}}{mc}$. The dynamics of the Φ -field is time-scale invariant, it does not depend on the arbitrarily chosen time lattice constant τ .

We can obtain a cubic mapping of type (9) in the following two very different situations. Either $\nu = O(1)$, i.e. $p_{max} = O(mc)$, and the potential parameters μ^2 and λ are of $O(1)$ as well. In this case we consider a low-energy model of vacuum fluctuations. We can, however, also let $p_{max} \rightarrow \infty$, thus considering a high-energy theory. In this case $\nu = \frac{p_{max}}{mc} \rightarrow \infty$. However, we can get *the same* finite cubic mapping if at the same time the parameters μ^2 and λ of the self-interacting potential diverge such that μ/ν and λ/ν remain finite.

There are distinguished parameter values leading to Tchebyscheff maps and thus to strongest possible chaotic behaviour. The negative 3rd-order Tchebyscheff map $\Phi_{n+1} = 3\Phi_n - 4\Phi_n^3$ is obtained from the potential

$$V_-^{(3)}(\Phi) = \nu(-\Phi^2 + \Phi^4)mc^2 + C_-, \quad (10)$$

the corresponding force is

$$F_-^{(3)}(\Phi) = \nu(2\Phi - 4\Phi^3) \frac{mc}{\tau}. \quad (11)$$

The positive 3rd-order Tchebyscheff map $\Phi_{n+1} = -3\Phi_n + 4\Phi_n^3$ is obtained from

$$V_+^{(3)}(\Phi) = \nu(2\Phi^2 - \Phi^4)mc^2 + C_+ \quad (12)$$

$$F_+^{(3)}(\Phi) = \nu(-4\Phi + 4\Phi^3)\frac{mc}{\tau}. \quad (13)$$

In fact, we can get any Tchebyscheff map $\pm T_N$ of order N , by considering appropriate potentials [2, 5].

Note that the strength of the potential is dependent on the energy scale $E_{max} = p_{max}c$ at which we look at the vacuum. This is indeed reasonable for a model of vacuum fluctuations. Namely, the potential should be proportional to the energy ΔE associated with a vacuum fluctuation, and from $\Delta E \Delta t = O(\hbar)$ we expect a larger energy on a smaller scale. Still the form of the dynamics in units of E_{max} is scale invariant. If a small coupling between neighbored phase space cells is introduced, approximate scale invariance is still retained. This is similar to velocity fluctuations in a fully developed turbulent flow, which are also approximately scale invariant and strongly chaotic.

We note that a Tchebycheff dynamics T_N with $N \geq 2$ produces information in each iteration step [14]. It is conjugated to a Bernoulli shift of N symbols. There is no time-reversal symmetry since each iterate Φ_n has N pre-images. Hence our chaotic dynamics describing vacuum fluctuations is something new, something in addition to the standard model field equations which do not have an arrow of time. We may associate the dynamics with a fundamental entropic field, whose main role is to produce information and to distinguish a particular direction of time.

4 Coulomb forces and Laplacian coupling

We may associate the strongly fluctuating variables p_n^i with the momenta of charged virtual particles that are created out of the self energy of the entropic field. For example, we may think of electrons and positrons, or any other types of fermions. Actually, we should think of a collective system of such charged particles, similar to a Dirac lake.

Suppose that (for example) Φ_n^i represents the momentum of a virtual electron and Φ_n^{i+1} the momentum of a neighbored virtual positron. The

Coulomb potential between two opposite charges at distance $r = |\vec{r}|$ is

$$V_{el}(r) = -\hbar c \alpha \frac{1}{r}. \quad (14)$$

$\alpha \approx 1/137$ is the fine structure constant. The force (= momentum exchange per time unit Δt) is

$$F_{el}(r) = \frac{\Delta p_{el}}{\Delta t} = -\frac{\partial}{\partial r} V_{el}(r) = -\hbar c \alpha \frac{1}{r^2} \quad (15)$$

$$\Longleftrightarrow \Delta p_{el} = -\hbar c \alpha \frac{\Delta t}{r^2}. \quad (16)$$

Again, due to the uncertainty relation $\Delta E \Delta t = O(\hbar)$ it does not make sense to choose an infinitesimally small time unit Δt . It is more reasonable to choose

$$c \Delta t = r \quad (17)$$

since photons move with the velocity of light. We then end up with the fact that the Coulomb potential gives rise to the momentum transfer

$$\Delta p_{el} = -\hbar \alpha \frac{1}{r} \quad (18)$$

during the time unit $\Delta t = r/c$.

In our picture of vacuum fluctuations, distances Δx and hence also inverse distances $1/r$ are strongly fluctuating due to the uncertainty relation. In section 2 we attributed the strongly fluctuating inverse distance variable $|\frac{\Gamma}{\hbar} p_n^i|$ to each phase space cell i . The maximum value of the inverse distance, corresponding to the smallest possible distance at a given energy scale p_{max} , is given by $\frac{\Gamma}{\hbar} p_{max}$. What should we now take for the inverse *interaction* distance between two neighbored particles i and $i+1$? Obviously, the relevant quantity is the momentum *difference* between them. According to the uncertainty relation, local momentum differences always correspond to local inverse distances between cells. Hence we define the inverse interaction distance $\frac{1}{r_{i,i+1}}$ between the electron in cell i and the positron in cell $i+1$ as the following strongly fluctuating random variable:

$$\frac{1}{r_{i,i+1}} = \frac{\Gamma p_{max}}{2\hbar} |\Phi_n^{i+1} - \Phi_n^i| \quad (19)$$

The factor $\frac{1}{2}$ is needed to let the inverse interaction distance not exceed the largest possible value $\frac{\Gamma}{\hbar} p_{max}$ of the inverse distance. It follows that the absolute value of the momentum transfer between cell i and cell $i + 1$ is

$$|\Delta p_{i,i+1}| = \hbar \alpha \frac{1}{r_{i,i+1}} = p_{max} \Gamma \frac{\alpha}{2} |\Phi_n^{i+1} - \Phi_n^i| \quad (20)$$

The momentum transfer can be positive or negative with equal probability, depending on whether we have equal or opposite charges in the neighbored cells. A possible choice for the signs is to take

$$\Delta p_{i,i+1} = p_{max} \Gamma \frac{\alpha}{2} (\Phi_n^{i+1} - \Phi_n^i) \quad (21)$$

This, indeed, causes inhomogeneities of the Φ -field to be smoothed out: If $\Phi^{i+1} > \Phi^i$, Φ^i increases. If $\Phi^i = \Phi^{i+1}$ (homogeneity), there is no change at all. If $\Phi^{i+1} < \Phi^i$, Φ^i decreases.

Similarly, the momentum transfer from the left neighbor is

$$\Delta p_{i,i-1} = p_{max} \Gamma \frac{\alpha}{2} (\Phi_n^{i-1} - \Phi_n^i) \quad (22)$$

Thus

$$\Delta p_{i+1,i} = p_{max} \Gamma \frac{\alpha}{2} (\Phi_n^i - \Phi_n^{i+1}) = -\Delta p_{i,i+1}. \quad (23)$$

The momentum exchange is antisymmetric under the exchange of the two particles, as it should be. We now have a direct physical interpretation associated with the signs of the Φ -field: If $\Phi_n^{i+1} > \Phi_n^i$, we have opposite charges in cell i and $i + 1$, causing attraction. Otherwise, the charges have equal signs, causing repulsion.

The above approach corresponds to diffusive coupling. However, we could also look at a momentum transfer given by $-p_{max} \Gamma \frac{\alpha}{2} (\Phi_n^{i-1} + \Phi_n^i)$, as generated by anti-diffusive coupling. In this case one assumes that the average momentum $\frac{1}{2}(\Phi_n^{i-1} + \Phi_n^i) p_{max}$ determines the inverse interaction distance $1/r_{i-1,i}$. Note that this approach still generates diffusive coupling if at the same time one electron is re-interpreted as a positron, which formally, according to Feynman, has a momentum of opposite sign. So both coupling forms can be physically relevant.

In total, the momentum balance equation for cell i is

$$p_{n+1}^i = p_n^i + \Delta p_{i,i-1} + \Delta p_{i,i+1} \quad (24)$$

$$\Longleftrightarrow \Phi_{n+1}^i = \Phi_n^i + \Gamma \frac{\alpha}{2} (\pm \Phi_n^{i-1} - 2\Phi_n^i \pm \Phi_n^{i+1}), \quad (25)$$

where the \pm sign corresponds to diffusive or anti-diffusive coupling, respectively. Remarkably, the momentum cutoff p_{max} drops out, and we end up with an evolution equation where only dimensionless quantities Φ , α and Γ enter. Notice that eq. (25) is a discretized diffusion equation with diffusion constant $\Gamma\alpha$. Also notice that Γ is just the constant of $O(1)$ in the uncertainty relation. Γ^{-1} is the size of the phase space cells in units of \hbar . If we *define* phase space cells to have size \hbar , this implies $\Gamma = 1$. Then the only relevant constant remaining is the coupling strength α of the $1/r$ -potential.

Finally, we have to combine the chaotic self-interaction with the diffusive interaction. Since, due to the uncertainty principle, our time variable is effectively discrete, both interactions must alternate. First, in each cell i there is a ‘spontaneous’ creation of momentum due to the self-interacting potential $V_{\pm}^{(N)}$:

$$\Phi_{n+1}^i = \pm T_N(\Phi_n^i). \quad (26)$$

Then, momenta of neighbored particles smooth out due to Coulomb interaction. Setting $\Gamma = 1$ we have

$$\Phi_{n+2}^i = \Phi_{n+1}^i + \frac{\alpha}{2}(\pm\Phi_{n+1}^{i-1} - 2\Phi_{n+1}^i \pm \Phi_{n+1}^{i+1}). \quad (27)$$

Combining eqs. (26) and (27) we obtain the coupled map lattice

$$\Phi_{n+2}^i = (1 - \alpha)T_N(\Phi_n^i) \pm \frac{\alpha}{2}(T_N(\Phi_n^{i-1}) + T_N(\Phi_n^{i+1})), \quad (28)$$

where T_N can be either the positive or negative Tchebyscheff map. What we obtained by our simple intuitive arguments is just the chaotic string dynamics introduced in [1, 2, 5] but now derived in a pedestrian, easy-going way. Our physical derivation implies that the coupling constant α can be identified with a standard model coupling constant.

5 Feynman webs

Let us now further work out our interpretation and proceed to a more detailed physical interpretation of the chaotic string dynamics. Remember that in this interpretation we regard Φ_n^i to be a fluctuating momentum component associated with a particle i at time n . Neighbored particles i and $i - 1$ exchange momenta due to the diffusive coupling.

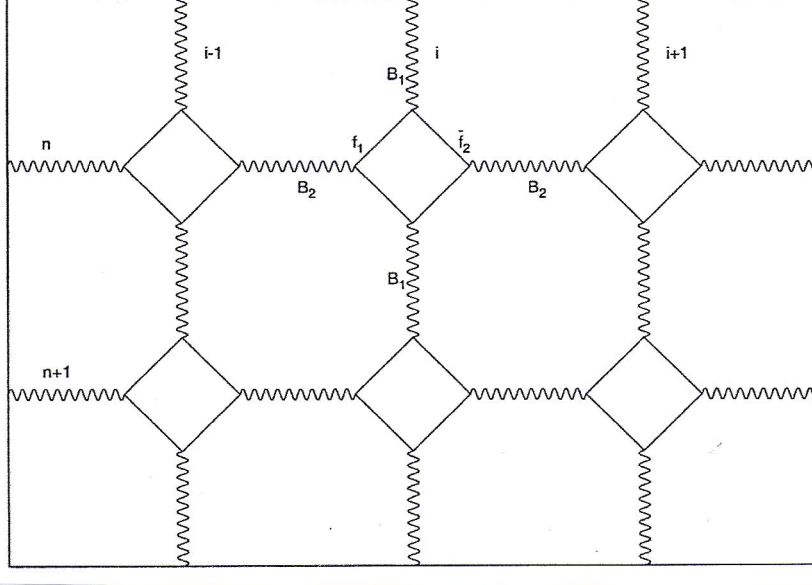


Figure 1: Feynman web interpretation of the coupled map dynamics.

A more detailed physical interpretation would be that at each time step n a fermion-antifermion pair f_1, \bar{f}_2 is being created in cell i by the field energy of the self-interacting fundamental entropic field. In units of some arbitrary energy scale p_{max} , the fermion has momentum Φ_n^i , the antifermion momentum $-\Phi_n^i$. They interact with particles in neighbored cells by exchange of a gauge boson B_2 , then they annihilate into boson B_1 and the next chaotic vacuum fluctuation (the next creation of a particle-antiparticle pair) takes place. This can be symbolically described by the Feynman graph in Fig. 1. Actually, the graph continues ad infinitum in time and space and could thus be called a ‘Feynman web’, since it describes an extended spatio-temporal interaction state of the entropic field, to which we have given a standard model-like interpretation. The important point is that in this interpretation α is a standard model coupling constant, since it describes the strength of momentum exchange of neighbored particles.

It is well known that standard model interaction strengths actually depend on the relevant energy scale E . We have the running electroweak and strong coupling constants [18]. For example, the fine structure constant $\alpha_{el}(E)$ slightly increases with E , and the strong coupling α_s rapidly decreases

with E . What should we now take for the energy (or temperature) E of the chaotic string? In [1, 2] extensive numerical evidence was presented that minima of the vacuum energy of the chaotic strings are observed for certain distinguished string couplings α_i , and these string couplings are numerically observed to coincide with running standard model couplings, the energy (or temperature) being given by

$$E = \frac{1}{2}N(m_{B_1} + m_{f_1} + m_{f_2}). \quad (29)$$

Here N is the index of the Tchebyscheff map of the chaotic string theory considered, and $m_{B_1}, m_{f_1}, m_{f_2}$ denote the masses of the particles involved in the Feynman web interpretation. The surprising observation is that rather than yielding just some unknown exotic physics, the chaotic string spectrum appears to reproduce the masses and coupling constants of the known quarks, leptons and gauge bosons of the standard model (plus possibly more).

Formula (29) formally reminds us of the zeropoint energy levels $E_N = \frac{N}{2}\hbar\omega$ of N quantum mechanical harmonic oscillators. In the Feynman web interpretation of Fig. 1, the formula is plausible. We expect the process of Fig. 1 to be possible as soon as the energy per cell i is of the order $m_{B_1} + m_{f_1} + m_{f_2}$. The boson B_2 is virtual and does not contribute to the energy scale. The factor N can be understood as a multiplicity factor counting the number of degrees of freedom. Given some value Φ_n^i of the momentum in cell i , there are N different pre-images $T_N^{-1}(\Phi_n^i)$ how this value of the momentum can be achieved. All these different channels contribute to the energy scale.

6 Heat bath of the vacuum

Let us now work out a statistical mechanics of the vacuum in somewhat more detail. We regard the vacuum as a kind of heat bath of virtual particles. Generally we want to use concepts from statistical mechanics and information theory. First, consider ordinary statistical mechanics. Given a system of N_p classical particles with Hamiltonian

$$H = \sum_{i=1}^{N_p} \frac{\vec{p}_i^2}{2m_i} + \frac{1}{2} \sum_{i,j} V(\vec{q}_i, \vec{q}_j) \quad (30)$$

the probability density ρ to observe a certain microstate $(\vec{q}_1, \dots, \vec{q}_{N_p}, \vec{p}_1, \dots, \vec{p}_{N_p}) := (q, p)$ is

$$\rho(q, p) = \frac{1}{Z(\beta)} e^{-\beta H(q, p)}. \quad (31)$$

We assume that Boltzmann statistics is applicable. $\beta = 1/kT$ is the inverse temperature and

$$Z(\beta) = \int d\vec{q}_1 \cdots d\vec{q}_{N_p} d\vec{p}_1 \cdots d\vec{p}_{N_p} e^{-\beta H(q, p)} \quad (32)$$

is the partition function. The internal energy U is defined as the expectation of H :

$$U = \langle H \rangle = \int d\vec{q}_1 \cdots d\vec{q}_{N_p} d\vec{p}_1 \cdots d\vec{p}_{N_p} \rho(q, p) H(q, p) = -\frac{\partial}{\partial \beta} \log Z(\beta) \quad (33)$$

If we want to develop a thermodynamic description of the vacuum we need a statistical theory of vacuum fluctuations Δq_i and Δp_i allowed by the uncertainty relation. The situation, however, is different from ordinary statistical mechanics because the momentum and position variables $\Delta q_i, \Delta p_i$ cannot be chosen independent from each other, as for ordinary statistical mechanics. If we choose a certain Δp_i then $\Delta q_i = \hbar/\Delta p_i$ is already fixed. Moreover, since virtual momenta violate energy conservation (they are just defined as doing that), we cannot expect to have an ordinary Hamiltonian $H(q, p)$ as in classical mechanics. If anything, the dynamics should be dissipative. This is why we base our statistical mechanics of the vacuum on the dissipative dynamics of section 2-4.

We also have to decide what ‘temperature’ means for the vacuum. It appears most reasonable to identify $kT \sim E$ with the energy scale E at which we look at the vacuum. Then $q_{min} = O(\frac{\hbar c}{kT})$ is the smallest spatial scale resolution we can achieve at this temperature, and $p_{max} = \frac{\hbar}{q_{min}}$ is the maximum momentum. As worked out in the previous sections, the relevant information on the state of a phase space cell of size \hbar is assumed to be given by a field variable Φ_n^i , which is the momentum uncertainty $\Delta p_i = p_n^i$ in units of p_{max} at time n in cell i . The corresponding position uncertainty is $x_n^i = \hbar/p_n^i$.

Since the vacuum is isotropic, the direction in which we measure the momentum is irrelevant. If there are d spatial directions, the d components $\Delta p_{x_1}, \dots, \Delta p_{x_d}$ of the momentum uncertainty into the d space directions are

expected to be independent from each other. In empty space, we do not expect any interactions between Δp_{x_1} and Δp_{x_2} for two different directions. Rather, 1-dimensional models are expected to do a good job.

We can either construct models where Φ_n^i is a pure random field, or where there is an underlying chaotic dynamics. The second type of models, which leads to chaotic strings in a natural way, has been shown [1, 2, 5, 9, 10] to reproduce observed standard model parameters (fermion and boson masses, coupling constants, mixing angles) with very high precision, taking as the leading principle the minimization of vacuum energy.

We do not have a true Hamiltonian for the chaotic string dynamics since the dynamics is dissipative. But we can write down a kind of analogue of a Hamiltonian given by

$$H = \sum_i V_{\pm}(\Phi^i) + aW_{\pm}(\Phi^i, \Phi^{i+1}), \quad (34)$$

with the self-interacting potential V given by eq. (12) (or its generalization) acting first, then followed by a potential $W \sim (\Phi^i - \Phi^{i+1})^2$ generating the diffusive coupling via nearest neighbor interaction. Due to the uncertainty relation, the time variable is effectively discrete with a lattice constant of order $\Delta t = \hbar/E$. In order to define an internal energy of vacuum fluctuations similar to eq. (33) we thus have to decide whether we relate it to V at one time step, to W at the next time step, or to the sum of both averaged over both time steps. These degrees of freedom are absent in classical statistical mechanics, where the time evolution is continuous. All three types of vacuum energies are important [2].

The equilibrium distributions, replacing the canonical probability distributions of ordinary statistical mechanics, are the invariant densities $\rho(\Phi^1, \Phi^2, \dots)$ of the coupled map dynamics. In contrast to ordinary statistical mechanics, there is no simple analytic expression for them, except for the uncoupled case $\alpha = 0$, where we have [14]

$$\rho(\Phi^1, \Phi^2, \dots) = \prod_i \frac{1}{\pi \sqrt{1 - \Phi^{i2}}}. \quad (35)$$

These types of densities can be dealt with in the formalism of nonextensive statistical mechanics [15, 16], they correspond to q -Gaussians with $q = 3$, respectively $q = -1$ if the escort formalism is used [14, 17]. Generally, the invariant densities depend on the coupling α in a non-trivial way. All averages

are formed with these densities. For ergodic systems the ensemble averages can be replaced by time averages.

Note that a dynamics generated by a Tchebyscheff map does not have a unique inverse, hence an arrow of time arises in a natural way. This arrow of time of the heat bath of the vacuum helps to justify the arrow of time in ordinary statistical mechanics, it is associated with our fundamental entropic field. Whereas classical mechanics is invariant under time reversal, the dynamics of the vacuum fluctuations considered here is not. In our approach the arrow of time enters at a fundamental level, as a hidden entropic dynamics of the vacuum.

7 States of maximum information and minimum correlation

Given the potential V of eq. (12), or its generalizations discussed in [5], the expectation $\langle V(\Phi) \rangle(a) =: V(a)$ measures the self energy of the vacuum per phase space cell (or per virtual particle). This can be regarded as a kind of thermodynamic potential of the vacuum. Numerically it is obtained by iterating the coupled map lattice (28) for a given coupling $\alpha = a$, choosing random initial conditions and averaging $V_+^{(N)}(\Phi_n^i)$ over all n and i (disregarding the first few transients). Numerical results for $V(a)$ were presented in detail in [1, 2]. It turns out that this function typically varies smoothly with a but has lots of local minima and maxima. What is the physical interpretation of such an extremum?

We may interpret $V(a)$ as a kind of entropy function of the vacuum. Clearly the Tchebyscheff maps, as any chaotic maps, produce information when being iterated. Or, looking at this the other way round, information on the precise initial value is lost in each iteration step due to the sensitivity on initial conditions [14]. The potential $V(\Phi)$ generates the chaotic dynamics and hence could be formally regarded as a kind of information potential. Its expectation measures the missing information (=entropy) we have on the particle contents of the phase space cells. At a minimum of $V(a)$ we have minimum missing information. In other words, we have maximum information on the particle contents of the cells. Hence we can associate the dynamics with a particular Feynman web at this point, and a should then coincide with the corresponding standard model coupling. This is what is

indeed observed, see [1, 2] for details.

Another interesting observable is the correlation function $C(a) = \langle \Phi^i \Phi^{i+1} \rangle$ of nearest neighbors. One observes that $C(a)$ typically varies smoothly with a , and that it vanishes at certain distinguished couplings $a_i \neq 0$ [1, 2, 12]. States of the vacuum with vanishing correlation are clearly distinguished — they describe, in a sense, a state where the system, although deterministic chaotic, is as random as possible. A zero of $C(a)$ means that the correlation between the momenta of neighbored virtual particles vanishes, meaning that we can clearly distinguish the particles in the various phase space cells, so that again a Feynman web with a definite particle contents makes sense. If a standard model coupling is chosen to coincide with a zero of the interaction energy, then this clearly represents a distinguished state of the heat bath of the vacuum, with a vanishing spatial 2-point function just as for uncoupled independent random variables, well suitable for stochastic quantization methods [19, 20].

We could interpret the correlation function as describing the polarization of the vacuum. Suppose, for example, that Φ_n^i represents a momentum component of an electron. Then, according to Feynman, $-\Phi_n^i$ could be interpreted as the momentum component of a positron. A negative correlation function $\langle \Phi_n^i \Phi_n^{i+1} \rangle$ means that if there is an electron in cell i , then with slightly larger probability there is a positron in cell $i + 1$, since the expectation of the product $\Phi_n^i \Phi_n^{i+1}$ is negative. A zero of the correlation function thus means the onset of vacuum polarization. Again we expect the threshold points where vacuum polarization sets in to occur at Feynman webs with energy $E = \frac{N}{2}kT$, with $kT = m_{B_1} + m_{f_1} + m_{f_2}$. Numerical evidence that stable zeros indeed coincide with running standard model coupling constants evaluated at these energy scales has been presented in [1, 2, 12].

8 Conclusion and Outlook

In this paper we have re-derived the chaotic string dynamics previously introduced in [1, 2] in a way that uses the language and tools of statistical mechanics. We associated the rapidly fluctuating chaotic dynamics with a fundamental entropic field that produces information and that can decay into virtual standard model particles. The description then basically reduces to a novel statistical mechanics description of the vacuum. The entropic field is responsible for the arrow of time in nature at a fundamental level.

Suitable thermodynamic potentials can then be defined and investigated as a function of the coupling parameter a . Numerical evidence presented in [1, 2, 5, 7, 9, 11, 12, 13] has shown that these generalized thermodynamic potentials distinguish standard model parameters. Note that the dynamics underlying our approach is discrete, nonlinear, chaotic, coupled and complex. This is complexity science at a fundamental level.

An interesting question for future research is whether it is possible to embed our statistical mechanics of the vacuum into some greater theory. Maybe the fundamental entropic field generating the chaotic dynamics is something that is really relevant before the creation of space-time, at a stage of the universe where information (rather than matter and radiation) is the relevant concept and standard model properties (including fundamental constants) are still in the process of being fixed. The space-time foam of string theory might yield another embedding scenario [21]. At the Planck scale, quantum coherence is likely to be broken due to spacetime foams and the Hawking effect, and there is clearly some sort of effective coarse graining going on at this scale, so that tools from statistical mechanics become more and more important.

Our derivation was based on classical mechanics, but relativity can emerge out of classical mechanics using superstatistical path integrals, as shown in [22]. Intrinsically, our statistical mechanics of the vacuum can indeed be related to the superstatistics concept [23], in the sense that the vacuum fluctuations of our model have non-Gaussian probability distributions which can be obtained by averaging over many inverse temperatures. These different inverse temperatures correspond to the various scales on which vacuum fluctuations take place. The superstatistical description then involves a fundamental symmetry, the Euclidean group in 1 dimension, as shown in [24], which is expected to play an important role for our coupled model of vacuum fluctuations as well.

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References

- [1] C. Beck, Physica D 171, 72 (2002)
- [2] C. Beck, Spatio-temporal Chaos and Vacuum Fluctuations of Quantized Fields, World Scientific (2002)
- [3] C.P. Dettmann, Physica D 172, 88 (2002)
- [4] C. Beck, in Proc. Idea-finding Symp. FIAS, EP Systema [hep-th/0305173]
- [5] C. Beck, Phys. Rev. D 69, 123515 (2004)
- [6] C.P. Dettmann and D. Lippolis, Chaos Sol. Fract. 23, 43 (2005)
- [7] S. Groote and C. Beck, Phys. Rev. E 74, 046216 (2006)
- [8] S. Groote and C. Beck, Dyn. Syst. 22, 219 (2007)
- [9] C. Beck, in ‘The logic of nature, complexity and new physics: From quark-gluon plasma to superstrings, quantum gravity and beyond, ed. A Zichichi, p.1-37, World Scientific (2008) [hep-th/0702082]
- [10] M.Maher and C. Beck, Chaos Sol. Fract. 37, 9 (2008)
- [11] S. Groote and H. Veermäe, Chaos Sol. Fract. 41, 2354 (2009)
- [12] M. Schaefer and C. Beck, Dyn. Syst. 25, 253 (2010)
- [13] M. Schaefer and M. Greiner, Chaos Sol. Fract. 44, 93 (2011)
- [14] C. Beck and F. Schlögl, Thermodynamics of Chaotic Systems, Cambridge University Press (1993)
- [15] C. Tsallis, J. Stat. Phys. 52, 479 (1988)
- [16] C. Tsallis, Introduction to Nonextensive Statistical Mechanics, Springer (2009)
- [17] C. Beck, Physica A 340, 459 (2004)
- [18] G. Kane, Modern Elementary Particle Physics, Addison-Wesley (1993)
- [19] G. Parisi and Y.S. Wu, Sci. Sin. 24, 483 (1981)
- [20] P.H. Damgaard and H. Hueffel, Stochastic quantization, Word Scientific (1988)
- [21] N.E. Mavromatos and S. Sarkar, Phys. Rev. D 79, 104015 (2009)
- [22] P. Jizba and H. Kleinert, Phys. Rev. D 82, 085016 (2010)

- [23] C. Beck and E.G.D. Cohen, *Physica A* 322, 267 (2003)
- [24] R. Hanel, S. Thurner, M. Gell-Mann, *Proc. Nat. Acad. Sci.* 108, 6390 (2011)